

Solving Equations with Rational Exponents
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You know a lot about inverses in mathematics – we use them every time we solve equations.

Write down the inverse operation for each of the following (there could be more than one correct answer) and then give a definition for “inverse” in your own words.

If you get stuck, it may be helpful for you to write the expression out and think what you would do to solve an equation that had that expression on one side of the equation.

The phrase...	Is the expression...	And its inverse is...
adding 5 to a number	$x + 5$	Subtracting 5 from a number
subtracting 7 from a number		
multiplying a number by $\frac{1}{2}$		
Multiplying a number by $\frac{2}{5}$		
dividing a number by 3		
squaring a number		
Taking the square root of a number		
Raising a number to the 5 th power		
Taking the 5 th root of a number		
Raising a number to the $\frac{2}{5}$ power		

An “inverse” is...

Make sure you have all of the answers correct to the table above before going on. Now that you’re finished, we’re going to combine our skills from this unit so far to solve some more challenging equations.

Skill	Partner A	Partner B
1	$\sqrt[4]{a} = 14$	$\sqrt[5]{b} = 50$
2	$\sqrt[5]{a^9} = 26$	$\sqrt[4]{b^3} = 27$
3	$(\sqrt[7]{a})^3 = 21$	$(\sqrt{b})^5 = 12$
4	$a^{\frac{1}{5}} = 50$	$b^{\frac{1}{4}} = 14$
5	$a^{\frac{3}{4}} = 27$	$b^{\frac{9}{5}} = 26$
6	$(a^{\frac{1}{2}})^5 = 12$	$(b^{\frac{1}{7}})^3 = 21$

Work with a partner – Decide who will be Partner A and who will be Partner B.

Compare the answers you got when you practiced Skills 1-3 with the answers you got when you practiced Skills 4-6. Work with your partner to explain your findings.

The previous problems only had one step. You cannot do this step until the radical or rational exponent is isolated on one side of the equation. You can isolate the radical using the inverses discussed at the beginning of the lesson. There are also some problems below in which the rational exponent or radical is applied to the entire side of the equation. Only in these situations will you undo the rational exponents or radicals first. Before solving the entire problem, make sure you know what the first step will be.

Step 1 $x^{1/4} - 2 = 3$

Step 1 $4x^7 - 6 = -2$

Step 1 $3(x^{2/3} + 5) = 207$

Step 1 $1450 = 800\left(1 + \frac{x}{12}\right)^{7.8}$

Step 1 $14.2 = 222.1 \cdot x^{3.5}$

Step 1 $3x^{3/4} + 5 = 53$

Step 1 $x^{1/2} - 5 = 0$

Step 1 $(2x + 7)^{1/2} = 3$

Step 1 $\sqrt[3]{x - 2} = 4$

Step 1 $\sqrt{a + 2} - 2 = 12$

Step 1 $\sqrt{2x - 5} = 9$

Step 1 $\sqrt[4]{3x + 1} - 5 = 0$

The next few problems are...different. We're going to come across some equations that have no solution and some that have two solutions. Remember, you can always check your answers by substituting your solution into the equation to make sure it works. In fact, you really ***need*** to check your answers to these problems! When we solve an equation correctly, but the answer doesn't work when we check it, we call the solution extraneous.

$$\sqrt{a+2} - 2 = a$$

$$\sqrt{3x-2} = -5$$

$$(2x+7)^{1/2} - x = 2$$

$$3x^{4/3} + 5 = 53$$

Applications of Equations with Rational Exponents or Radicals.
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The distance between two points is $5\sqrt{2}$. If one of the points is located at $(4,2)$ and the other point has a x-value of -1 , what are the possible y-values of the other point?

The volume of a sphere is 2145. If the formula $V = \frac{4}{3}\pi r^3$ is used to calculate the volume of a sphere, what is the radius of the sphere?

The equation $v = \sqrt{2.5r}$ allows you to calculate the maximum velocity, v , that a car can safely travel around a curve with a radius of r feet. This is used by the Department of Transportation to determine the best speed limit for a given stretch of road. If a road has a speed limit of 45 mph, what is the tightest turn on that road?