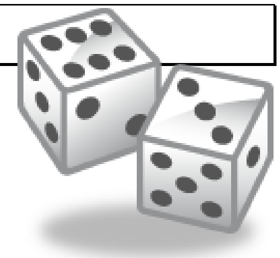
Common Core Math II Name Date

Investigation : Exponential Growth & Decay



Materials Needed:

- Graphing Calculator (to serve as a random number generator)

To use the calculator's random integer generator feature:

1. Type any number besides zero into your calculator, press [STO→] , [MATH], [←] , [ENTER], [ENTER]

2. Press [MATH] [←] [5] [1] [,] [6] [)]

You can use numbers other than 1 and 6 here. The calculator will choose numbers between and including these numbers when you press enter. Continue pressing enter for more numbers.

Investigation:

1. Choose a recorder to collect the class's data on the board. You'll copy the data down in your table later.
2. Everyone in the class should stand so that the recorder can count everyone and record the number of people standing in a table for "Stage 0".
3. Use your calculator to find a random integer between 1 and 6. If you roll a 1, sit down. Before proceeding, allow time for the recorder to count the number of people still standing. When the recorder is finished counting, (s)he will let you know.
4. Repeat step 3 until fewer than 3 people are standing (or you run out of room on the table).
5. Record the data in your table.

Stage	0	1	2	3	4	5	6	7	8	9	10
People Standing											

Questions:

1. What is your initial value for this set of data? What does it represent in the investigation?

2. Would it make more sense to find a common ratio (r) or common difference (d) for this data? Explain.

3. Based on your answer to Question 2, find the r OR d for the data you collected. Show the process you used to do so.

4. Could you estimate your answer to Question 3 without conducting the exploration? If so, how?

5. Write a recursive (NOW-NEXT) function that would help you make predictions for this data.

6. Write an explicit function using function notation that would help you make predictions for this data. In your function let x be the stage of the investigation and let f(x) equal the number of people standing in that stage.

Investigation 2:
Half of a radioactive substance decays every 53 years. How much will remain of a 12 milligram sample after 530 years?

Complete the table.

Years	0	53	106	159	212	265	318	371	424	477	530
Remaining radioactive substance											

Questions:

7. What is your initial value for this set of data? What does it represent in the investigation?

8. Would it make more sense to find a common ratio (r) or common difference (d) for this data? Explain.

9. Based on your answer to Question 8, find the r OR d for the data you collected. Show the process you used to do so.

10. Could you estimate your answer to Question 9 without filling in the table? If so, how?

11. Write a recursive (NOW-NEXT) function that would help you make predictions for this data.

12. Write an explicit function using function notation that would help you make predictions for this data. In your function let x be the number of 53 year increments in the investigation and let f(x) equal the amount of radioactive substance remaining.

13. Write an explicit function using function notation that would help you make predictions for this data. In your function let x be the number of years in the investigation and let f(x) equal the amount of radioactive substance remaining. Use your equation to determine how much radioactive substance will remain after 500 years.

14. When will there be exactly 5 milligrams of the radioactive substance? Determine your answer to the nearest month.

15. Compare Investigation 1 and Investigation 2. What are the similarities and differences?

Investigation 3:
You invest money in a savings account that earns 2.5% interest annually. How much money will you have at the end of 10 years if you begin with $1000?

Complete the table.

Years	0	1	2	3	4	5	6	7	8	9	10
Money in your account											

Questions:

16. What is your initial value for this set of data? What does it represent in the investigation?

17. Would it make more sense to find a common ratio (r) or common difference (d) for this data? Explain.

18. Based on your answer to Question 17, find the r OR d for the data you collected. Show the process you used to do so.

19. Could you estimate your answer to Question 18 without filling in the table? If so, how?

20. Write a recursive (NOW-NEXT) function that would help you make predictions for this data.

21. Write an explicit function using function notation that would help you make predictions for this data. In your function let x be the number of years in the investigation and let f(x) equal the amount of money in the account.

22. You find a bank that will pay you 3% interest annually, so you consider moving your account. Your current bank decides you're a good customer and offers you a special opportunity to compound your interest semiannually!!! (They make it sound like it's a really good deal, so you're curious). You don't play around with your money, so you ask what exactly that means. They explain that you'll still get 2.5% interest, but they'll give you 1.25% interest at the end of June and 1% interest at the end of December. You want to see if you make more money than you would if you just switched banks, so you do the calculations. Which bank is giving you a better deal? Explain your answer.

When you write an equation for a situation and use it to make predictions, you assume that other people who use it will understand the situation as well as you do. That's not always the case when you take away the context, so we sometimes need to provide some additional information to accompany the equation.

23. The domain of a function is the set of all the possible input values that can be used when evaluating it. If you remove your functions in Questions 6 and 13 and 21 from the context of this situation and simply look at the table and/or graph of the function, what numbers are part of the theoretical domain of the function?

    Will this be the case with all exponential functions? Why or why not?

24. When you consider the context, however, not all of the numbers in the theoretical domain really make sense. We call the numbers in the theoretical domain that make sense in our situation the practical domain. For instance, in the first investigation, our input values are "Stages". If you look at the tables you created, what numbers would be a part of the practical domain for these investigations?

    Investigation 1

    Investigation 2

    Investigation 3

    Make note of any similarities and differences and explain why they exist.

Review of Vocabulary

- Initial Value: The first term in a sequence or the dependent value associated with an independent value of 0.
- Common Ratio: The ratio of one term in a sequence and the previous term.
- Common Difference: The difference between one term in a sequence and the previous term.
- Recursive function: a function that can be used to find any term in a sequence if you have the previous term (i.e. NOW-NEXT)
- Explicit function: a function that can be used to find any term in the sequence without having to know the previous term. (i.e. y = or f(x) =)
- Domain: the set of all input values for a function
- Theoretical Domain: the set of all input values for a function without consideration for context
- Practical Domain: the set of all input values for a function that are reasonable within context

Summary of Findings

The results of Investigation 1 & 2 simulate exponential decay because the values are being multiplied by the same value between 0 and 1 each time. The results of Investigation 3 simulate exponential growth because the values are being multiplied by the same value greater than 1 each time.

As you may have noticed in Questions 4, the common ratio is related to the probability that you will be randomly assigned a 1 out of 6 options. Since this probability is 1:6, or $\frac{1}{6}$, or $0.1\overline{6}$, or $16.\overline{6}\%$ that fraction of the class will sit down. These numbers do not represent our common ratio, however. That's because when 1 of 6 people sit down, that leaves 5:6, or $\frac{5}{6}$ or $0.8\overline{3}$ or 83% of the class still standing. To find the common ratio (b) using this probability (p) or any percentage for that matter, you can use the following equations. The probability or percent MUST be written as a decimal to use this formula.

Exponential Growth	Exponential Decay
$b = \|1 + p\|$ This is because you begin with 100% (1 when written as a decimal) and add the same percentage each time.	$b = \|1 - p\|$ This is because you begin with 100% (1 when written as a decimal) and subtract the same percentage each time.

In problems 5, 11, and 20, you wrote recursive functions for the exponential data. Make sure you include the starting value of your data (a) – this will always be where the independent variable is 0.

$NEXT = NOW \cdot b$, Starting at a $\qquad\qquad$ $y = ab^x$

For each of these Investigations, we came up with functions with theoretical domains of all real numbers. In fact, all exponential functions will have domains of all real numbers. However, when we considered the context, we realized that the practical domain for these the first investigation only involves the whole numbers because all of the other real numbers would not work as "stages". We also saw that in most scenarios where the domain is related to time, the domain is only positive values.