

# Investigation: Theoretical vs. Experimental Probability

## Part 1: Theoretical Probability

Probability is the chance or likelihood of an event occurring. We will study two types of probability, theoretical and experimental.

**Theoretical Probability:** the probability of an event is the ratio or the number of favorable outcomes to the total possible outcomes.

$$P(\text{Event}) = \frac{\text{Number or favorable outcomes}}{\text{Total possible outcomes}}$$

**Sample Space:** The set of all possible outcomes. For example, the sample space of tossing a coin is {Heads, Tails} because these are the only two possible outcomes. Theoretical probability is based on the set of all possible outcomes, or the sample space.

1. List the sample space for rolling a six-sided die (remember you are listing a set, so you should use brackets {}):

Find the following probabilities:

P(2)

P(3 or 6)

P(odd)

P(not a 4)

P(1,2,3,4,5, or 6)

P(8)

2. List the sample space for tossing two coins:

Find the following probabilities:

P(two heads)

P(one head and one tail)

P(head, then tail)

P(all tails)

P(no tails)

3. Complete the sample space for tossing two six-sided dice:

{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6),  
(2,1), (2,2), (2,\_\_\_), \_\_\_\_, \_\_\_\_, \_\_\_\_,  
(3,1), \_\_\_\_, \_\_\_\_, \_\_\_\_, \_\_\_\_, \_\_\_\_,  
\_\_\_\_, \_\_\_\_ , \_\_\_\_ , \_\_\_\_ , \_\_\_\_ , \_\_\_\_ ,  
\_\_\_\_ , \_\_\_\_ , \_\_\_\_ , \_\_\_\_ , \_\_\_\_ , \_\_\_\_ ,  
\_\_\_\_ , \_\_\_\_ , \_\_\_\_ , \_\_\_\_ , \_\_\_\_ , \_\_\_\_ }

Find the following probabilities:

P(a 1 and a 4)

P(a 1, then a 4)

P(sum of 8)

P(sum of 12)

P(doubles)

P(sum of 15)

4. When would you expect the probability of an event occurring to be 1, or 100%? Describe an event whose probability of occurring is 1.

5. When would you expect the probability of an event occurring to be 0, or 0%? Describe an event whose probability of occurring is 0.

## Part 2: Experimental Probability

**Experimental Probability:** the ratio of the number of times the event occurs to the total number of trials.

$$P(\text{Event}) = \frac{\text{Number of times the event occurs}}{\text{Total number of trials}}$$

1. Do you think that theoretical and experimental probabilities will be the same for a certain event occurring? Explain your answer.

2. Roll a six-sided die and record the number on the die. Repeat this 9 more times

Number on Die	Tally	Frequency
1		
2		
3		
4		
5		
6		
<b>Total</b>		10

Based on your data, find the following experimental probabilities:

P(2)

P(3 or 6)

P(odd)

P(not a 4)

How do these compare to the theoretical probabilities in Part 1? Why do you think they are the same or different?

3. Record your data on the board (number on die and frequency only). Compare your data with other groups in your class. Explain what you observe about your data compared to the other groups. Try to make at least two observations.

4. Combine the frequencies of all the groups in your class with your data and complete the following table:

Number on Die	Frequency
1	
2	
3	
4	
5	
6	
<b>Total</b>	

Based on the whole class data, find the following experimental probabilities:

P(2)

P(3 or 6)

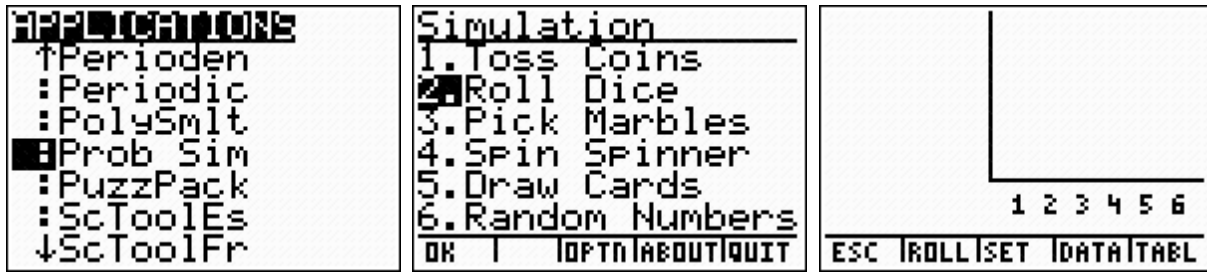
P(odd)

P(not a 4)

How do these compare to your group's probabilities? How do these compare to the theoretical probabilities from Part 1?

What do you think would happen to the experimental probabilities if there were 200 trials? 500 trials? 1000 trials? 1,000,000 trials?

5. On your graphing calculator, go to APPS and open Prob Sim. Press any key and then select 2: Roll dice.



Click Roll. Notice that there will be a bar on the graph at the right. What does this represent?

Now push +1 nine more times. Push the right arrow to see the frequency of each number on the die. How many times did you get a 1? \_\_\_\_\_ A 2? \_\_\_\_\_ A 5?

Now press the +1, +10, and +50 buttons until you have rolled 100 times. Based on the data, find the following experimental probabilities:

P(2)                      P(3 or 6)                      P(odd)                      P(not a 4)

Press the +50 button until you have rolled 1000 times. Based on the data, find the following experimental probabilities:

P(2)                      P(3 or 6)                      P(odd)                      P(not a 4)

Press the +50 button until you have rolled 5000 times. Based on the data, find the following experimental probabilities:

P(2)                      P(3 or 6)                      P(odd)                      P(not a 4)

What can you expect to happen to the experimental probabilities in the long run? In other words, as the number of trials increases, what happens to the experimental probabilities?

Why can there be differences between experimental and theoretical probabilities in general?

### Part 3: Which one do I use?

So when do we use theoretical probability or experimental probability? Theoretical probability is always the best choice, when it can be calculated. But sometimes it is not possible to calculate theoretical probabilities because we cannot possibly know all of the possible outcomes. In these cases, experimental probability is appropriate. For example, if we wanted to calculate the probability of a student in the class having green as his or her favorite color, we could not use theoretical probability. We would have to collect data on the favorite colors of each member of the class and use experimental probability.

Determine whether theoretical or experimental probability would be appropriate for each of the following. Explain your reasoning:

1. What is the probability of someone tripping on the stairs today between first and second periods?
2. What is the probability of rolling a 3 on a six-sided die, then tossing a coin and getting a head?
3. What is the probability that a student will get 4 of 5 true false questions correct on a quiz?
4. What is the probability that a student is wearing exactly four buttons on his or her clothing today?